

()

: /

()

()

:

Z Y X

X	59,61,62,58,60
Y	50,60,66,54,70
Z	39,65,46,78,72

()

()

.Measures of Dispersion

-

:

()

: /

- = R

.

= -

= -

:

R : ()

82, 40, 62, 70, 30, 80

=

= :

R = 82 - 30 = 52

:

R : ()

	40-49	50-59	60-69	70-79	80-89	90-99
	2	9	15	11	2	1

:

:

, =

, =

:

R = 94.5 - 44.5 = 50

, =

:

, =

R = 99.5 - 39.5 = 60

:

-

()

-

-

:

n

:

Q_n _____ Q_2 _____ Q_n

$\frac{n}{4}$

Q_1

$\frac{3n}{4}$

Q_3

Q_3 Q_1

Q

:

$Q = \frac{Q_3 - Q_1}{2},$ (1)

$Q_2 = \frac{n}{2}$

Q_2

Q

:

-

Q_1

-

Q_3

-

()

: /

()

-

:

:()

67,65,69,58,55,71,72,70

:

:

55,58,65,67,69,70,71,72

$$Q_1 = \frac{58+65}{2} = 61.5,$$

$$Q_3 = \frac{70+71}{2} = 70.5$$

$$Q = \frac{Q_3 - Q_1}{2} = 4.5$$

:

:()

59,67,65,69,58,55,70,72,74

:

:

55,58,59,65,67,69,70,72,74

$$Q_1 = 59,$$

$$Q_3 = 70$$

$$Q = \frac{Q_3 - Q_1}{2} = 5.5$$

$$\frac{n}{2}$$

$$\frac{n}{4}$$

Q_1

$$\frac{n}{2}$$

$$\frac{3n}{4}$$

Q_3

:

Q_3 Q_1

()

$$Q_1 = A_1 + \frac{(\frac{n}{4} - f_1)}{f_2 - f_1} L, \quad (2)$$

$$Q_3 = A_2 + \frac{(\frac{3n}{4} - f_1')}{f_2' - f_1'} L, \quad (3)$$

.()

:()

:

:

< 39.5	0
< 49.5	2
< 59.5	11
< 69.5	26
< 79.5	37
< 89.5	39
< 99.5	40

$n = 40,$ $\frac{n}{4} = 10,$ $\frac{3n}{4} = 30,$ $L = 10$

$$Q_1 = 49.5 + \left(\frac{10 - 2}{11 - 2} \right) 10$$

$$Q_1 = 49.5 + 8.89 = 58.39$$

$$Q_3 = 69.5 + \left(\frac{30 - 26}{37 - 26} \right) 10$$

$$Q_3 = 69.5 + 3.64 = 73.14$$

$$Q = \frac{Q_3 - Q_1}{2} = 7.38$$

:

-

-

:

-

-

-

\bar{x}

:

M.D.

$$M.D. = \frac{1}{n} \sum_{i=1}^n |x_i - \bar{x}|, \quad (4)$$

() .()

:

$$\text{M.D.} = \frac{\sum_{i=1}^n f_i |x_i - \bar{x}|}{n}, \quad (5)$$

n

:()

6,5,7,7,8,9,9,5

:

:

x	$x - \bar{x}$	$ x - \bar{x} $
6	-1	1
5	-2	2
7	0	0
7	0	0
8	1	1
9	2	2
9	2	2
5	-2	2
56	0	10

$$\bar{x} = \frac{\sum x}{n}$$

$$\bar{x} = \frac{56}{8} = 7$$

$$\text{M.D.} = \frac{10}{8} = 1.25,$$

. ()

: ()

:

:

Classes	x	f	$f x$	$x - \bar{x}$	$ x - \bar{x} $	$ x - \bar{x} f$
40-49	44.5	2	89	-21.25	21.25	42.5
50-59	54.5	9	490.5	-11.25	11.25	101.25
60-69	64.5	15	967.5	-1.25	1.25	18.75
70-79	74.5	11	819.5	8.75	8.75	96.75
80-89	84.5	2	169	18.75	18.75	37.5
90-99	94.5	1	94.5	28.75	28.75	28.75
		40	2630			325

$$\bar{x} = \frac{\sum fx}{n}$$

$$\bar{x} = \frac{2630}{40} = 65.75$$

$$\text{M.D.} = \frac{325}{40} = 8.125,$$

:

()

σ^2

σ

:

N

\bar{X}

X_1, X_2, \dots, X_N

:

$$(X_1 - \bar{X})^2, (X_2 - \bar{X})^2, \dots, (X_N - \bar{X})^2$$

: σ^2

$$\sigma^2 = \frac{1}{N} \sum_{i=1}^N (X_i - \bar{X})^2, \quad (6)$$

:

$$\sigma = \sqrt{\frac{1}{N} \sum_{i=1}^N (X_i - \bar{X})^2}, \quad (7)$$

()

n .()

S^2 S

: $(n-1)$

$$S^2 = \frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2, \quad (8)$$

$$S = \sqrt{\frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2}, \quad (9)$$

S

σ^2 S^2 ()

:

8,9,7,6,5

:()

:

x	$x - \bar{x}$	$(x - \bar{x})^2$
8	1	1
9	2	4
7	0	0
6	-1	1
5	-2	4
35	0	10

$$\bar{x} = \frac{\sum x}{n}$$

$$\bar{x} = \frac{35}{5} = 7$$

$$S^2 = \frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2,$$

$$S^2 = \frac{1}{5-1}(10) = 2.5$$

$$S = \sqrt{2.5} = 1.581,$$

: ()

$$S^2 = \frac{1}{n-1} \left(\sum_{i=1}^n x_i^2 - \frac{(\sum_{i=1}^n x_i)^2}{n} \right), \quad (10)$$

:

$$S^2 = \frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2$$

$$S^2 = \frac{1}{n-1} \sum_{i=1}^n (x_i^2 - 2x_i\bar{x} + \bar{x}^2)$$

$$S^2 = \frac{1}{n-1} \left(\sum_{i=1}^n x_i^2 - 2\bar{x} \sum_{i=1}^n x_i + n\bar{x}^2 \right)$$

$$S^2 = \frac{1}{n-1} \left(\sum_{i=1}^n x_i^2 - n\bar{x}^2 \right)$$

$$S^2 = \frac{1}{n-1} \left(\sum_{i=1}^n x_i^2 - \frac{(\sum_{i=1}^n x_i)^2}{n} \right)$$

$$\cdot \sum x^2 \quad \sum x \quad ()$$

. ()

()

:()

:

:

x	x^2
8	64
9	81
7	49
6	36
5	25
35	255

$$S^2 = \frac{1}{n-1} \left(\sum_{i=1}^n x_i^2 - \frac{(\sum_{i=1}^n x_i)^2}{n} \right)$$

$$S^2 = \frac{1}{4} \left(255 - \frac{(35)^2}{5} \right) = 2.5$$

$$S = \sqrt{2.5} = 1.581,$$

()

c

:

:

d_1, d_2, \dots, d_n

x_1, x_2, \dots, x_n

$$d_n = x_n + c, \dots, d_2 = x_2 + c, d_1 = x_1 + c$$

$$\therefore S^2 = \frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2$$

$$S^2 = \frac{1}{n-1} \left(\sum_{i=1}^n [(d_i \pm c) - (\bar{d} \pm c)]^2 \right) = \frac{1}{n-1} \sum_{i=1}^n (d_i - \bar{d})^2, \tag{11}$$

:

c ()

:()

:

:

x	$d = x - 5$	d^2
8	3	9
9	4	16
7	2	4
6	1	1
5	0	0
	10	30

$$S^2 = \frac{1}{n-1} \left(\sum_{i=1}^n d_i^2 - \frac{(\sum_{i=1}^n d_i)^2}{n} \right)$$

$$S^2 = \frac{1}{4} \left(30 - \frac{(10)^2}{5} \right) = 2.5$$

$$S = \sqrt{2.5} = 1.581,$$

()

:

:

c

x_1, x_2, \dots, x_n

d_1, d_2, \dots, d_n

$$d_1 = cx_1, d_2 = cx_2, \dots, d_n = cx_n$$

$$\begin{aligned}
S_x^2 &= \frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2 \\
S_x^2 &= \frac{1}{n-1} \sum_{i=1}^n \left(\frac{d_i}{c} - \frac{\bar{d}}{c}\right)^2 = \frac{1}{n-1} \frac{1}{c^2} \sum_{i=1}^n (d_i - \bar{d})^2 \\
S_x^2 &= \frac{1}{c^2} S_d^2 \Rightarrow S_x = \frac{1}{c} S_d, \tag{12}
\end{aligned}$$

()

$$S_x = c S_d, \tag{13}$$

.()

\bar{x}

$\bar{x} \neq a$ a

$$\begin{aligned}
\sum (x-a)^2 &= \sum (x + \bar{x} - \bar{x} - a)^2 \\
&= \sum [(x - \bar{x}) + (\bar{x} - a)]^2 \\
&= \sum (x - \bar{x})^2 + n(\bar{x} - a)^2 + 2(\bar{x} - a) \sum (x - \bar{x}) \\
&= \sum (x - \bar{x})^2 + n(\bar{x} - a)^2
\end{aligned}$$

$$n(\bar{x} - a)^2$$

$$\sum (x-a)^2 < \sum (x-\bar{x})^2$$

$$S_2^2 \quad S_1^2 \quad n_2 \quad n_1$$

$$S^2 = \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 1}$$

$x_1, x_2, \dots, x_{n_1}, y_1, y_2, \dots, y_{n_2}$

$$S_1^2 = \frac{1}{n_1 - 1} \sum_{i=1}^{n_1} (x_i - \bar{x})^2$$

$$S_2^2 = \frac{1}{n_2 - 1} \sum_{i=1}^{n_2} (y_i - \bar{x})^2$$

$$(n_1 - 1) S_1^2 = \sum_{i=1}^{n_1} (x_i - \bar{x})^2$$

$$(n_2 - 1) S_2^2 = \sum_{i=1}^{n_2} (y_i - \bar{y})^2$$

$$(n_1 - 1) S_1^2 + (n_2 - 1) S_2^2 = \sum_{i=1}^{n_1} (x_i - \bar{x})^2 + \sum_{i=1}^{n_2} (y_i - \bar{y})^2 = \sum_{i=1}^{n_1+n_2} (z_i - \bar{x})^2$$

$$\therefore S^2 = \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 1}$$

)

:

.(

f_1, f_2, \dots, f_k x_1, x_2, \dots, x_k k
 : () () ()

$$S^2 = \frac{1}{n-1} \sum_{i=1}^n f_i (x_i - \bar{x})^2, \quad S = \sqrt{\frac{1}{n-1} \sum_{i=1}^n f_i (x_i - \bar{x})^2}, \quad (14)$$

$$S^2 = \frac{1}{n-1} \left(\sum_{i=1}^n f_i x_i^2 - \frac{(\sum_{i=1}^n f_i x_i)^2}{n} \right), \quad (15)$$

$$S^2 = \frac{1}{n-1} \left(\sum_{i=1}^n f_i d_i^2 - \frac{(\sum_{i=1}^n f_i d_i)^2}{n} \right), \quad (16)$$

:

()

()

:()

() ()

:

()

- :

Classes	x	f	xf	$x - \bar{x}$	$(x - \bar{x})^2$	$(x - \bar{x})^2 f$
40-49	44.5	2	89	-21.25	451.56	903.13
50-59	55.5	9	490.5	-11.25	126.56	1139.06
60-69	65.5	15	967.5	-1.25	1.56	23.44
70-79	75.5	11	819.5	8.75	76.56	842.19
80-89	85.5	2	169	18.75	351.56	703.13
90-99	95.5	1	94.5	28.75	826.56	826.56
Total		40	2630			4437.5

$$\bar{x} = \frac{1}{n} \sum_{i=1}^k f_i x_i = \frac{1}{40} (2630) = 65.75$$

$$S^2 = \frac{1}{n-1} \sum_{i=1}^n f_i (x_i - \bar{x})^2 = \frac{1}{40-1} (4437.5) = 113.78$$

$$S = 10.67,$$

: ()

Classes	x	f	xf	$x^2 f$
40-49	44.5	2	89	3960.5
50-59	55.5	9	490.5	26732.25
60-69	65.5	15	967.5	62403.75
70-79	75.5	11	819.5	61052.75
80-89	85.5	2	169	12280.5
90-99	95.5	1	94.5	8930.25
Total		40	2630	177360

$$S^2 = \frac{1}{n-1} \left(\sum_{i=1}^n f_i x_i^2 - \frac{(\sum_{i=1}^n f_i x_i)^2}{n} \right) = \frac{1}{40-1} (177360 - 172922.5) = 113.78$$

$$S = 10.67, \quad ()$$

$$c = 64.5 \quad () \quad ()$$

: ()

Classes	x	f	$d = x - 64.5$	df	$d^2 f$
40-49	44.5	2	-20	-40	800
50-59	55.5	9	-10	-90	900
60-69	65.5	15	0	0	0
70-79	75.5	11	10	110	1100
80-89	85.5	2	20	40	800
90-99	95.5	1	30	30	900
Total		40		50	4500

$$S^2 = \frac{1}{n-1} \left(\sum_{i=1}^n f_i d_i^2 - \frac{(\sum_{i=1}^n f_i d_i)^2}{n} \right) = \frac{1}{40-1} (4500 - 62.5) = 113.78$$

$$S = 10.67,$$

:

)

: (

Classes	x	f	$d = x - 64.5$	$d/10 = d'$	$d'f$	$d'^2 f$
40-49	44.5	2	-20	-2	-4	8
50-59	55.5	9	-10	-1	-9	9
60-69	65.5	15	0	0	0	0
70-79	75.5	11	10	1	11	11
80-89	85.5	2	20	2	4	8
90-99	95.5	1	30	3	3	9
Total		40			5	45

$$S_{d'}^2 = \frac{1}{n-1} \left(\sum_{i=1}^n f_i d_i'^2 - \frac{(\sum_{i=1}^n f_i d_i')^2}{n} \right) = \frac{1}{40-1} (45 - 0.625) = 1.1378$$

$$S_{d'} = 1.067, \quad S_x = 10 S_{d'} = 10.67 \quad ()$$

\bar{x} x_1, x_2, \dots, x_n X
 : Z s
 $Z_i = \frac{x_i - \bar{x}}{s}, \quad i = 1, 2, \dots, n$
 " Z_i
 . () "
 : ()

$$Z_1 = \frac{82 - 75}{10} = 0.7$$

$$Z_2 = \frac{89 - 81}{16} = 0.5$$

1- $r = \frac{3(\bar{x} - Med)}{s}$, (17)

2- $r = \frac{\bar{x} - Mod}{s}$, (18)

3- $r = \frac{m_3^2}{s^3}$, (19)

$$m_3 = \frac{\sum (x - \bar{x})^3}{n},$$

$$m_3 = \frac{\sum f(x - \bar{x})^3}{n},$$

() : ()

$\bar{x} = 65.75$, $Med = 65.41$, $Mod = 65.5$, $s = 10.67$

$r = \frac{3(\bar{x} - Med)}{s} = \frac{3(65.75 - 65.41)}{10.67} = 0.069$, (1)

: ()

$$r = \frac{\bar{x} - Mod}{s} = \frac{65.75 - 65.5}{10.67} = 0.025, \quad (2)$$

-

: k

$$k = \frac{m_4}{s^4} - 3$$

:

$$m_4 = \frac{\sum (x - \bar{x})^4}{n},$$

$$m_4 = \frac{\sum f(x - \bar{x})^4}{n},$$

6,3,5,5,9,4,6,7,1,2,4,8

	58-60	61-63	64-66	67-69	70-72	73-75
	2	7	14	15	8	4

. ()

	< 10	10-14	15-19	20-24	25-29	30≤
	5	20	35	19	13	8

2, 5, 9, 4, 3, 6

70, 70, 70, 70, 70, 70, 70